PHYS158: Introductory Physics for Engineers II
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21 Electric Charge and Electric Field

21.1 Electric Charge

electric charge (q): something on amber after rubbing it with wool
elektron is Greek for "amber". After rubbing fur on a plastic rod and silk on a glass rod, the fur and the glass rod have **positive** charge while the silk and the plastic rod have **negative** charge.
electrostatics: interactions between electric charges at rest
Like-signed charges repel and opposite-signed charges attract.
atom: made up of negative **electrons**, positive **protons**, and neutral **neutrons**
ionization: the gain or loss of electrons from an atom
ion: an atom that has gained a net electric charge because its number of protons and electrons are not equal; most electric phenomena are due to either free electrons in metals or ions
conservation of charge: the sum of all electric charges in any closed system is constant
quantization of charge: all electric charges are integer multiples of the charge of an electron

21.2 Conductors, Insulators, and Induced Charges

conductors: materials that allow charge to flow through them, including most metals
insulators: materials that do not allow charge to flow through them, including most nonmetals
induced charges: net charges created by redistributing the charge in an body
induction: the process of charging a conductor, where free electrons move to one side
polarization: the process of charging an insulator, where polar molecules shift to one orientation

21.3 Coulomb's Law

point charge: idealized charged bodies whose charge is wholly concentrated in a single point
Coulomb (C): the SI unit of electric charge, $1\text{ C} = 1\text{ A}\cdot\text{s}$
The fundamental unit of electric charge is the magnitude e of the charge of an electron:

$$e = 1.602176487(40) \times 10^{-19}\text{ C} \tag{1}$$

permittivity of free space (ϵ_0): the proportionality constant used in all electricity equations when there are no dielectrics or when only considering microscopic bodies
Coulomb's law: the electrostatic force is an inverse-square relationship
The force $\mathbf{F}$ between two point charges q_1 and q_2 separated by displacement vector $\mathbf{r}$ is:

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} \\ \mathbf{F} &= \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^3}\mathbf{r} \end{aligned} \tag{2}$$

Vectors $\mathbf{v}$ are bolded and their corresponding magnitudes v are not.
superposition of forces: the net electric force is the vector sum of the individual electric forces

Coulomb's constant (k): the proportionality constant between **F** and q_1q_2/r^2 in vacuum

$$\begin{aligned} k &= 8.987551787 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \\ &= \frac{1}{4\pi\epsilon_0} \\ \epsilon_0 &= 8.854187818 \times 10^{-12} \text{ C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2} \end{aligned} \tag{3}$$

21.4 Electric Field and Electric Forces

electric field (**E**): the field of a charged body, which exerts forces on other charged bodies
test charge (q_0): the point charge that is used to theoretically define electric fields
The electric field at a point is the force $\mathbf{F}_0$ experienced by a unit test charge at that point:

$$\mathbf{E} = \frac{\mathbf{F}_0}{q_0} \tag{4}$$

The electric field on the **field point** from a point charge q at the **source point** acts along the displacement vector **r** from the source point to the field point:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \mathbf{r} \tag{5}$$

Electric field is in general a **vector field**, but it can have the same values, or be **uniform**, within a certain region. In electrostatics, the electric field in a conductor is always zero.

21.5 Electric-Field Calculations

superposition of fields: the total field is the vector sum of the individual fields
linear charge density λ: the charge per unit length
surface charge density σ: the charge per unit area
volume charge density ρ: the charge per unit volume

21.6 Electric Field Lines

field line: an imaginary line whose tangents at any point are in the direction of the field
field map: a two-dimensional cross section of a field, showing its field lines
Field lines can never intersect, and the density of field lines is proportional to the field strength.

21.7 Electric Dipoles

cross product ($\times$): the operator that outputs a vector perpendicular to both input vectors
dot product ($\cdot$): the operator that outputs a number related to the angle of the input vectors

$$\begin{aligned} \|\mathbf{u} \times \mathbf{v}\| &= uv \sin\theta \\ \mathbf{u} \cdot \mathbf{v} &= uv \cos\theta \end{aligned} \tag{6}$$

right hand rule: the procedure to find the direction of the cross product: if the first operand points with your right index and the second operand points with your right middle, then the cross product points with your right thumb; and if there is a tendency to rotate along the direction that your right hand fingers curl, then the moment points with your right thumb
electric dipole: a pair of equal and opposite point charges q with displacement **d** along the dipole axis from the negative charge to the positive charge

electric dipole moment ($\mathbf{p}$): the product of the charge and displacement of an electric dipole

$$\mathbf{p} = q\mathbf{d} \tag{7}$$

The net force on an electric dipole in a uniform electric field is zero, but not the net torque:

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \tag{8}$$

Electric fields can induce electric dipoles in neutral bodies in the same direction, which is why neutral bodies are attracted by both positively and negatively charged bodies.
The electric field distance r away from an infinite sheet of uniform charge is proportional to 1; an infinite line of charge is proportional to $1/r$; a single point charge is proportional to $1/r^2$; a dipole is proportional to $1/r^3$; and two dipoles or a quadrupole is proportional to $1/r^4$.

22 Gauss's Law

22.1 Charge and Electric Flux

closed surface: a surface that completely encloses a volume
flux (Φ): the amount of field that flows through an area
flux density: the amount of flux that flows across an area per unit area; another name for field
Sign conventions: the electric field and electric field lines created by a positive point charge points away from the charge; the force on a positive test charge points in the direction of the electric field; and the electric flux through a closed surface enclosing a positive charge is positive.

22.2 Calculating Electric Flux

For a uniform electric field through a flat surface, the electric flux Φ_E depends on the electric field $\mathbf{E}$ and vector area $\mathbf{A} = A\mathbf{n}$, where A is the surface area and $\mathbf{n}$ is the unit normal vector that always points outwards from a closed surface:

$$\Phi_E = \mathbf{E} \cdot \mathbf{A} \tag{9}$$

In the general case, the electric flux Φ_E through a closed surface is the surface integral:

$$\Phi_E = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} \tag{10}$$

22.3 Gauss's Law

The examples in the textbook often use ϵ_0 and μ_0, but these are replaced here by ϵ and μ.
Use ϵ_0 if a dielectric constant is not and μ_0 if a relative permeability is not explicitly given.
Gauss's law: the flux through any closed surface is proportional to the net charge Q_{encl} inside:

$$\Phi_E = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q_{encl}}{\epsilon} \tag{11}$$

Gaussian surface: any imaginary closed surface used in Gauss's law

22.4 Applications of Gauss's Law

The electric fields at a distance r away from the centre of these bodies with net charge q are:

$$\begin{aligned}
E &= \frac{1}{4\pi\epsilon}\frac{q}{r^2} && \text{spherical or point charge} \\
E &= \frac{1}{4\pi\epsilon}\frac{qr}{R^3} && \text{inside uniformly charged spherical insulator with radius } R \\
E &= \frac{1}{2\pi\epsilon}\frac{\lambda}{r} && \text{infinite cylinder or wire with charge density } \lambda \\
E &= \frac{\sigma}{2\epsilon} && \text{infinite sheet with charge density } \sigma \\
E &= \frac{\sigma}{\epsilon} && \text{oppositely charged conducting plates} \\
E &= 0 && \text{inside any conductor}
\end{aligned} \tag{12}$$

22.5 Charges on Conductors

The excess charges in a conductor only reside on its surface, including the surfaces of its cavities if some charge is placed inside the cavity. Charges in an insulator can be anywhere.
The electric field inside a conductor is zero. The electric field at the surface of a conductor is perpendicular to the surface and depends on the surface charge density σ:

$$\mathbf{E}\cdot\mathbf{n} = \frac{\sigma}{\epsilon} \tag{13}$$

23 Electric Potential

23.1 Electric Potential Energy

conservative force: a force such that the work done by it is path-independent
potential energy (U): the energy in a body due to its position
The work W done by any force $\mathbf{F}$ on a particle that moves from point a to point b is a line integral, but if the force is conservative, can also be expressed as a change in potential energy:

$$W = \int_a^b \mathbf{F}\cdot d\mathbf{l} = U_a - U_b = -\Delta U \tag{14}$$

The potential energy of a dipole $\mathbf{p}$ in an electric field $\mathbf{E}$ is at a minimum in stable equilibrium:

$$U = -\mathbf{p}\cdot\mathbf{E} \tag{15}$$

Since electrostatic fields are conservative, if $U = 0$ when they are infinitely far apart, then the electric potential energy with a test charge q_0 a distance r away from a point charge q is:

$$U = \frac{q_0}{4\pi\epsilon_0}\frac{q}{r} \tag{16}$$

23.2 Electric Potential

potential (V): the potential energy associated with a unit test charge at some position

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{17}$$

Volt (V): the SI unit of electric potential, $1\ \mathrm{V} = 1\ \mathrm{J \cdot C^{-1}}$
voltage (V_{ab}): the difference in potential between point a and point b

$$V_{ab} = V_a - V_b \tag{18}$$

Since electrostatic fields are conservative, integrating the field $\mathbf{E}$ along any path gives the voltage:

$$V_{ab} = \int_a^b \mathbf{E} \cdot \mathrm{d}\mathbf{l} \tag{19}$$

electron volt (eV): the change in potential energy of an electron moving through one volt

$$1\ \mathrm{eV} = 1.602176487(40) \times 10^{-19}\ \mathrm{J} \tag{20}$$

23.3 Calculating Electric Potential

dielectric breakdown: the process of an insulator becoming a conductor due to an electric field, which is strong enough to ionize the molecules in the insulator
corona: the current and associated glow through air after its dielectric breakdown

23.4 Equipotential Surfaces

equipotential surface: a surface on which potential is constant
Field lines are always normal to equipotential surfaces, and equipotential surfaces can never intersect. When all charges are at rest, the surface of a conductor is an equipotential surface, so its entire solid volume is at the same potential.

23.5 Potential Gradient

gradient ($\boldsymbol{\nabla}$): the operator that outputs a vector whose components are the partial derivatives of the respective components of the input vector

$$\mathbf{E} = -\boldsymbol{\nabla} V \tag{21}$$

24 Capacitance and Dielectrics

24.1 Capacitors and Capacitance

capacitor: two conductors insulated from each other
The conductors of a capacitor are often called plates. A capacitor with charge Q has charge $+Q$ on plate a and $-Q$ on plate b, resulting in a potential difference V_{ab} across it.
capacitance (C): the proportionality constant between Q and V_{ab}

$$C = \frac{Q}{V_{ab}} \tag{22}$$

Farad (F): the SI unit of capacitance, $1\ \mathrm{F} = 1\ \mathrm{C \cdot V^{-1}}$
parallel-plate capacitor: two parallel conducting plates with area A separated by distance d
The capacitance depends only on the shapes, sizes, separation, and stuff between the conductors. For a parallel-plate capacitor where $d \ll A$:

$$C = \epsilon \frac{A}{d} \tag{23}$$

24.2 Capacitors in Series and in Parallel

equivalent capacitance (C_{eq}): the capacitance of a single capacitor with the same properties as a combination of capacitors
series connection: the connection of elements at two different voltages along the same path
Capacitors connected in series all have the same charge and their voltages add together:

$$\begin{aligned} Q &= Q_1 = Q_2 = \cdots = Q_n \\ V &= V_1 + V_2 + \cdots + V_n \\ \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} \end{aligned} \tag{24}$$

parallel connection: the connection of elements on two different paths across the same voltage
Capacitors connected in parallel all have the same voltage and their charges add together:

$$\begin{aligned} Q &= Q_1 + Q_2 + \cdots + Q_n \\ V &= V_1 = V_2 = \cdots = V_n \\ C_{eq} &= C_1 + C_2 + \cdots + C_n \end{aligned} \tag{25}$$

24.3 Energy Storage in Capacitors and Electric-Field Energy

The change in potential energy between an uncharged and a fully charged capacitor is:

$$\Delta U = \frac{1}{2} C V^2 \tag{26}$$

Applications of capacitors include condenser microphones, camera flash units, radio receivers, stud finders, pacemakers, touch screens, and nuclear fusion experiments.
energy density (u): the energy per unit volume of a space
The potential energy in any capacitor is stored in the electric field of the space between its plates:

$$u_E = \frac{1}{2} \epsilon E^2 \tag{27}$$

24.4 Dielectrics

dielectric: the insulator between a capacitor's plates
Dielectrics help separate plates and increase tolerance to dielectric breakdown.
electrometer: a device that measures the voltage between two conductors
dielectric constant or relative permittivity (K): the proportionality constant between the capacitance with a dielectric C and the capacitance in vacuum C_0
Polarization of a dielectric reduces the field between the capacitor plates, raising the capacitance:

$$\begin{aligned} E &= \frac{E_0}{K} \quad V = \frac{V_0}{K} \\ K &= \frac{C}{C_0} \qquad K \geq 1 \end{aligned} \tag{28}$$

dielectric strength (E_m): the maximum electric field a dielectric can withstand before dielectric breakdown, per unit thickness
All capacitors have maximum voltage ratings based on their dielectric strength and thickness; applying a larger voltage than the maximum rating risks permanently damaging the capacitor.

permittivity (ϵ): the proportionality constant between E and surface charge density σ
The permittivity with a dielectric is always greater than the permittivity of free space:

$$\epsilon = K\epsilon_0 \tag{29}$$

24.5 Molecular Model of Induced Charge

free charges: charges that are free to move
bound charges: charges that are bound to a molecule
Induced surface charges appear in conductors because free charges undergo induction.
Induced surface charges appear in dielectrics because bound charges redistribute themselves in an electric field, making all molecules into electric dipoles, which undergo polarization.

24.6 Gauss's Law in Dielectrics

Gauss's law only applies to free charges; all bound charges should be ignored.

25 Current, Resistance, and Electromotive Force

25.1 Current

current (I): the amount of charge flowing through an area per unit time

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{30}$$

conventional current: the amount of positive charge flowing through an area per unit time
Sign conventions: current is always conventional current, which in the absence of non-electrostatic forces always flows in the direction of the electric field from higher to lower potential.
Current in a wire always flows along the length of the wire. The direction of positive current must be defined; negative current flows in the opposite direction.
Ampere (A): the SI fundamental unit of current
drift velocity ($\mathbf{v}_d$): the net velocity of many charges
concentration (n): the number of moving charges per unit volume
Current depends on cross-sectional area A and the magnitude of each charge $|q|$:

$$I = n|q|v_d A \tag{31}$$

current density ($\mathbf{J}$): the current in the direction of the drift velocity per unit area

$$\begin{aligned} J &= \frac{I}{A} \\ \mathbf{J} &= nq\mathbf{v}_d \end{aligned} \tag{32}$$

25.2 Resistivity

Ohm's law: electric field is often directly proportional to current density
resistivity (ρ): the proportionality constant between $\mathbf{E}$ and $\mathbf{J}$

$$\rho = \frac{E}{J} \tag{33}$$

conductivity: the reciprocal of resistivity
ohmic or linear: describes a conductor that obeys Ohm's law
non-ohmic or nonlinear: describes a conductor that does not obey Ohm's law
As temperature T increases, the resistivity of most conductors increases.
temperature coefficient of resistivity (α): the proportionality constant between ρ and T
If ρ_0 is the resistivity at temperature T_0, then ρ at temperature T can be approximated by:

$$\rho = \rho_0 \left[1 + \alpha \left(T - T_0\right)\right] \tag{34}$$

25.3 Resistance

resistance (R): the proportionality constant between V and I
The resistance of an ohmic conductor with length L and cross-sectional area A is constant:

$$R = \frac{V}{I} = \frac{\rho L}{A} \tag{35}$$

Ohm (Ω): the SI unit of resistance, $1\,\Omega = 1\,\mathrm{V} \cdot \mathrm{A}^{-1}$
resistor: a circuit element designed to have a specific resistance

25.4 Electromotive Force and Circuits

complete circuit: something in which a steady current can flow
electromotive force or emf ($\mathcal{E}$): the energy per unit charge, supplied by a non-electrostatic force, that moves charges from lower to higher potential
emf source: a circuit element that provides emf
terminals: the two conductors of a circuit element that connect it to the circuit
terminal voltage (V_{ab}): the positive voltage between the terminals of an emf source
internal resistance (r): the resistance inside an emf source
A complete circuit must be a closed conducting loop with at least one emf source. The two terminals of an emf source are marked + at the higher potential and − at the lower potential. In a complete circuit with current I, resistance R, and only one emf source, the terminal voltage of the emf source is less than the emf because of its internal resistance:

$$V_{ab} = \mathcal{E} - Ir = IR \tag{36}$$

circuit diagram: a symbolic representation of a complete circuit

25.5 Energy and Power in Electric Circuits

The rate at which energy flows into or out of a circuit element with terminal voltage V_{ab} is:

$$P = V_{ab} I \tag{37}$$

Current always enters resistors at a higher potential, so energy is dissipated as heat.
Current usually leaves sources of emf at a higher potential, so energy is delivered as electric potential energy. If two sources of emf are connected, the source with the smaller emf stores energy from the other source, and also dissipates energy because of its internal resistance:

$$P = \left(\mathcal{E} + Ir\right) I \tag{38}$$

25.6 Theory of Metallic Conduction

mean free time (τ): the average time between collisions for one particle
Treating a group of electrons as classical particles of mass m, charge e, concentration n, and mean free time τ, whose average velocity $\mathbf{v}_d = 0$ in the absence of electric fields, then if the electric field exerts a force $\mathbf{F} = q\mathbf{E}$ on each electron:

$$\rho = \frac{m}{ne^2\tau} \tag{39}$$

As temperature increases, the mean free time decreases, so the resistivity of most conductors increases. As the temperature increases, the concentration of semiconductors increases very rapidly, so the resistivity of semiconductors decreases.

26 Direct-Current Circuits

26.1 Resistors in Series and in Parallel

direct-current circuits: circuits in which the current does not change direction
network: a complete circuit with many possible paths
equivalent resistance (R_{eq}): the resistance of a single resistor with the same properties as a combination of resistors
Resistors connected in series all have the same current and their voltages add together:

$$\begin{aligned} I &= I_1 = I_2 = \cdots = I_n \\ V &= V_1 + V_2 + \cdots + V_n \\ R_{eq} &= R_1 + R_2 + \cdots + R_n \end{aligned} \tag{40}$$

Resistors connected in parallel all have the same voltage and their currents add together:

$$\begin{aligned} I &= I_1 + I_2 + \cdots + I_n \\ V &= V_1 = V_2 = \cdots = V_n \\ \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \end{aligned} \tag{41}$$

26.2 Kirchhoff's Rules

junction: a point where three or more conductors meet
Kirchhoff's junction rule: the algebraic sum of the currents into any junction is zero
loop: any closed conducting path
Kirchhoff's loop rule: the algebraic sum of the voltages in any loop is zero
These arise from the conservation of electric charge and the conservative nature of electric fields.
Sign conventions: assume a direction for the current in every branch of the network, then travel around the loop and add voltages as you go. For sources of emf, voltage is $\mathcal{E}$ when travelling from $-$ to $+$. For resistors, voltage is IR when travelling against the current. For capacitors, voltage is Q/C when discharging.

26.3 Electrical Measuring Instruments

d'Arsonval galvanometer: a device that measures the current through its terminals
A galvanometer or meter is a pointer and spring attached to a coil of wire in the magnetic field of a permanent magnet. The deflection of the pointer is proportional to the current through it.
full-scale deflection: the maximum deflection of the meter pointer
The resistance of the meter is R_c, but the current required for full-scale deflection I_{fs} is tiny.
shunt resistor: a resistor connected in parallel that redirects some current
ammeter: a device that measures the current through its terminals
An ideal ammeter has no resistance and no voltage across it. Ammeters are connected in series to the circuit they measure and consist of a meter connected in parallel to a shunt resistor with resistance R_{sh}. The current I_a at full-scale deflection is:

$$(I_a - I_{fs})\, R_{sh} = I_{fs} R_c \tag{42}$$

voltmeter: a device that measures the voltage between its terminals
An ideal voltmeter has infinite resistance and no current through it. Voltmeters are connected in parallel to the circuit they measure and consist of a meter connected in series to a resistor with resistance R_s. The voltage V_v at full-scale deflection is:

$$V_v = I_{fs}\,(R_c + R_s) \tag{43}$$

ohmmeter: a device that measures the resistance between its terminals
Ohmmeters are connected in parallel to the circuit they measure and consist of an emf source, a meter, and a resistor connected in series, chosen so that $R = 0$ at full-scale deflection.
potentiometer: a device that measures the emf of an emf source; or, any variable resistor
Potentiometers consist of a uniform resistance wire and an emf source, connected through a sliding contact to a meter and the emf source they measure. If there is no deflection in the meter, then the emf of the unknown source is proportional to the position of the contact.
multimeter: a device that measures the voltage, current, and resistance between its terminals

26.4 *R*-*C* Circuits

R-*C* **circuit**: a circuit that includes a resistor and a capacitor
time constant or relaxation time (τ): the time it takes for a time-varying quantity to reach $(1 - 1/e)$ of its maximum value or fall to $(1/e)$ of its maximum value
Steady quantities V are capitalized and their corresponding time-varying quantities v are not.
If x is a time-varying quantity, τ is its time constant, and X_0 is its maximum value:

$$x = X_0 \left(e^{-t/\tau}\right) \tag{44}$$

$$x = X_0 \left(1 - e^{-t/\tau}\right) \tag{45}$$

When charging the capacitor, use (45) for q and (44) for i. When discharging the capacitor, use (44) for q and the negative of (44) for i. In a *R*-*C* circuit charged by an emf source $\mathcal{E}$, the time constant τ, maximum current I_0, and maximum capacitor charge Q_0 are:

$$\begin{aligned} \tau &= RC \\ I_0 &= \frac{\mathcal{E}}{R} \\ Q_0 &= C\mathcal{E} \end{aligned} \tag{46}$$

26.5 Power Distribution Systems

In household and automotive power distribution systems, all loads are connected in parallel to the power source so that shutting one off does not shut them all off. The power source provides two conductors, which are split up into individual branch circuits with outlets connected in parallel across the **hot side** and the **neutral side**, which is connected to ground.
Household voltage in Canada and the United States is 120 V; most of Europe use 240 V.
The resistance of the wires limits the maximum current available, which is 20 A for 12-gauge wire.
Special components are used on the hot side to interrupt circuits before exceeding the maximum current: **fuses** are resistors with a low melting temperature, which melt before exceeding the maximum, and **circuit breakers** are more expensive devices that act as reusable fuses.
Contact between the hot and neutral side creates a **short circuit**, which has such low resistance that the high current could ignite the wire without a fuse or circuit breaker. A broken wire creates a **open circuit**, which can cause intermittent sparking. In all modern devices, a **grounding wire** connects the metal frame of the device to ground. It normally carries no current, but if the hot side contacts the frame, it provides a current path to blows the fuse and grounds the frame.

27 Magnetic Field and Magnetic Forces

27.1 Magnetism

permanent magnet: a fragment of iron ore that exerts forces on other pieces of iron
magnetic pole: something in a permanent magnet
Magnesia is where the first permanent magnets were discovered. If a bar-shaped permanent magnet is free to rotate, the end that points north is a **north pole** and the end that points south is a **south pole**. Like-signed poles repel and opposite-signed poles attract.
The earth is a permanent magnet, and its north geographic pole is near its south magnetic pole.
magnetic inclination: the angle that the earth's magnetic field makes with the surface
magnetic declination: the deviation between the earth's magnetic axis and its axis of rotation
magnetic monopole: a single isolated magnetic pole, never yet experimentally observed
If a permanent magnet is broken in two, each broken end becomes a new pole.

27.2 Magnetic Field

magnetic field (**B**): the field of a moving charge, which exerts forces on other moving charges
The magnetic force **F** experienced by a charge q moving at a velocity **v** is:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{47}$$

Tesla (T): the SI unit of magnetic field, $1\ \text{T} = 1\ \text{N} \cdot \text{A}^{-1} \cdot \text{m}^{-1}$
The magnetic field can be measured using a moving test charge by orienting its movement to be perpendicular to the magnetic field and then measuring the force it experiences.

27.3 Magnetic Field Lines and Magnetic Flux

Sign conventions: the magnetic field of a permanent magnet outside it points away from the north pole and towards the south pole; the force on a north pole points in the direction of the magnetic field; and the magnetic flux through a surface enclosing a north monopole is positive.
To draw magnetic field lines in three dimensions, a dot $\cdot$ represents a vector directed out of the plane and a cross $\times$ represents a vector directed into the plane.

Weber (Wb): the SI unit of magnetic flux: $1\,\mathrm{W}b = 1\,\mathrm{T}\cdot\mathrm{m}^2$
The magnetic flux Φ_B through a closed surface is the following surface integral. But since all magnets observed are dipoles, the total magnetic flux through a closed surface is always zero:

$$\Phi_B = \oint \mathbf{B}\cdot \mathrm{d}\mathbf{A} = 0 \tag{48}$$

27.4 Motion of Charged Particles in a Magnetic Field

Since magnetic forces always act perpendicular to velocity, magnetic fields can never do work. Any work that magnetic fields seem to do is actually caused by induced electric fields.
Magnetic fields make charges move in a helix of radius R and angle ϕ with angular speed ω:

$$\begin{aligned} R &= \frac{mv\sin\phi}{|q|B} \\ \omega &= \frac{|q|B}{m} \end{aligned} \tag{49}$$

27.5 Applications of Motion of Charged Particles

Applications of the motion of charged particles include cyclotrons, magnetrons, magnetic bottles, bubble chambers, velocity selectors, mass spectrometers, and Thomson's experiment to measure the ratio of electron charge to mass.

27.6 Magnetic Force on a Current-Carrying Conductor

The magnetic force experienced by a straight conductor carrying a current I in a direction $\mathbf{l}$ is:

$$\mathbf{F} = I\mathbf{l}\times\mathbf{B} \tag{50}$$

27.7 Force and Torque on a Current Loop

magnetic dipole: a flat closed conducting loop with current I and vector area $\mathbf{A}$
magnetic dipole moment ($\boldsymbol{\mu}$): the product of the current and vector area of a magnetic dipole
The net force on a loop of current in a uniform magnetic field is zero, but not the net torque:

$$\begin{aligned} \boldsymbol{\mu} &= I\mathbf{A} \\ \boldsymbol{\tau} &= \boldsymbol{\mu}\times\mathbf{B} \end{aligned} \tag{51}$$

The potential energy U for a dipole in an magnetic field is at a minimum in stable equilibrium:

$$U = -\boldsymbol{\mu}\cdot\mathbf{B} \tag{52}$$

solenoid: a helical winding of wire, which amplifies magnetic effects by its number of turns N

$$\boldsymbol{\mu} = NI\mathbf{A} \tag{53}$$

Permanent magnets are magnetic dipoles, and magnetic dipoles create nonuniform magnetic fields, whose strength decreases with distance and thus create a net force along the direction of the field on other magnetic dipoles, which is why like-signed poles repel and opposite-signed poles attract. Magnetic fields can induce magnetic dipoles in nonmagnetic iron bodies in the same direction, which is why nonmagnetic iron bodies are attracted by either pole.

27.8 The Direct-Current Motor

Direct-current motors have a loop of wire in a uniform magnetic field which rotates because the commutator and brushes reverse the direction of its current every half-turn, thus converting electric energy into mechanical energy.

27.9 The Hall Effect

In a strip along the xz-plane with current density $\mathbf{J}$ in the x-direction and a uniform magnetic field $\mathbf{B}$ in the y-direction, both positive and negative charge carriers are pushed in the same z-direction to create a electric field $\mathbf{E}$ and corresponding **Hall voltage** in the z-direction.
If q is the charge of each carrier and n is the concentration of carriers:

$$nq = \frac{-J_x B_y}{E_z} \tag{54}$$

Moving the strip in the opposite direction of the current causes the Hall voltage to disappear when the speed is equal to the drift speed.

28 Sources of Magnetic Field

28.1 Magnetic Field of a Moving Charge

permeability of free space μ_0: the proportionality constant used in all magnetism equations when there are no magnetized bodies or when only considering microscopic bodies
A point charge q moving at a velocity $\mathbf{v}$ creates a magnetic field whose value depends on the displacement vector $\mathbf{r}$ from the source point to the field point:

$$\mathbf{B} = \frac{\mu}{4\pi} \frac{q}{r^3} (\mathbf{v} \times \mathbf{r}) \tag{55}$$

The metre and ampere are defined so that μ_0, ϵ_0, k, and the speed of light c are exactly:

$$\begin{aligned} c &= 2.99792458 \times 10^8 \text{ m} \cdot \text{s}^{-1} \\ c^2 &= \frac{1}{\epsilon_0 \mu_0} \\ k &= \frac{1}{4\pi\epsilon_0} = \left(10^{-7} \text{ N} \cdot \text{s}^2 \cdot \text{C}^{-2}\right) c^2 \\ \mu_0 &= 4\pi \times 10^{-7} \text{ T} \cdot \text{m} \cdot \text{A}^{-1} \end{aligned} \tag{56}$$

Sign conventions: point the thumb of your right hand in the direction of magnetic moment of a magnetic dipole, velocity of a positive charge, current in a straight conductor, magnetic field in a conducting loop, or current in any conductor. Now the fingers of your right hand curl around in the direction of current in the magnetic dipole, magnetic field of the charge, magnetic field of the conductor, current of the loop, or the direction of integration in Ampere's law, respectively.

28.2 Magnetic Field of a Current Element

Biot–Savart's law: the magnetic field from a conductor with current I in a direction $\mathrm{d}\mathbf{l}$ is given by integrating infinitesimal charges moving at a constant drift velocity:

$$\mathbf{B} = \frac{\mu}{4\pi} \int \frac{I}{r^3} (\mathrm{d}\mathbf{l} \times \mathbf{r}) \tag{57}$$

28.3 Magnetic Field of a Straight Current-Carrying Conductor

$$B = \frac{\mu I}{2\pi r} \tag{58}$$

28.4 Force Between Parallel Conductors

Two parallel straight conductors attract each other if their currents flow in the same direction:

$$\frac{F}{L} = \frac{\mu I_1 I_2}{2\pi r} \tag{59}$$

28.5 Magnetic Field of a Circular Current Loop

If a is the radius of the loop and x is the distance on the axis of the loop away from its centre:

$$\mathbf{B} = \frac{\mu}{2\pi\left(x^2 + a^2\right)^{3/2}}\boldsymbol{\mu} \tag{60}$$

28.6 Ampere's Law

Ampere's law: if current and electric field are all constant, then the line integral of magnetic field $\mathbf{B}$ in a closed loop is proportional to the net current I_{encl} inside it:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu I_{encl} \tag{61}$$

28.7 Applications of Ampere's Law

The magnetic field of a straight current-carrying conductor and a circular current loop are above. The field near the middle of a solenoid with n turns per unit length is 0 outside and $\mu n I$ inside.

28.8 Magnetic Materials

If e is the charge, m mass, and L angular momentum of an electron, then its magnetic moment is quantized by the Bohr magneton μ_B since its minimum L is the reduced Planck constant:

$$\begin{aligned} \mu &= \frac{e}{2m}L \\ \mu_B &= \frac{e}{2m}\frac{h}{2\pi} = 9.274 \times 10^{-24}\ \mathrm{A} \cdot \mathrm{m}^2 \end{aligned} \tag{62}$$

magnetization ($\mathbf{M}$): the net magnetic moment per unit volume V of a material

$$\mathbf{M} = \frac{\Sigma \boldsymbol{\mu}}{V} \tag{63}$$

Curie's law: paramagnetic magnetization is inversely proportional to absolute temperature T

$$\mathbf{M} = C\frac{\mathbf{B}}{T} \tag{64}$$

If a magnetized material completely surrounds a current-carrying conductor, then the total magnetic field in the material increases based on its magnetization:

$$\mathbf{B} = \mathbf{B}_0 + \mu_0 \mathbf{M} \tag{65}$$

relative permeability (K_m): the proportionality constant between $\mathbf{B}$ and $\mathbf{B}_0$
magnetic susceptibility (χ_m): the amount by which relative permeability differs from unity

$$\chi_m = K_m - 1 \tag{66}$$

permeability (μ): the proportionality constant used in all magnetism equations
The permeability of a magnetized material is always greater than the permeability of free space:

$$\mu = K_m \mu_0 \tag{67}$$

paramagnetism: the property of materials whose atoms have a non-negligible magnetic moment and whose magnetic susceptibility is positive, and so are affected by magnetic fields
diamagnetism: the property of materials whose atoms have no magnetic moment and whose magnetic susceptibility is negative, and so are not affected by magnetic fields
ferromagnetism: the property of materials whose atoms have a non-negligible magnetic moment which line up in magnetic domains, and so can become permanent magnets
hysteresis: the behaviour of ferromagnetic materials in which some magnetization remains even without any external magnetic field, so that more energy is lost when fully demagnetizing them

29 Electromagnetic Induction

29.1 Induction Experiments

induced current: the current created by a changing magnetic flux
induced emf or back emf ($\mathcal{E}$): the emf required to cause an induced current

29.2 Faraday's Law

Faraday's law of induction: the induced emf in a closed loop depends on the rate of change of magnetic flux through the loop

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \tag{68}$$

Sign conventions: point the thumb of your right hand in the direction of the normal vector of the loop. Now the fingers of your right hand curl around in the direction of positive induced emf.

29.3 Lenz's Law

Lenz's law: the magnetic field created by an induced current opposes the change that caused it

29.4 Motional Electromotive Force

motional electromotive force: the induced emf created by moving a closed conducting loop in an unchanging magnetic field
Moving the loop at velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ creates an electric field with an electric force that cancels the magnetic force on its charges, so that the total emf is given by integrating the force per unit charge $\mathbf{E} = \mathbf{v} \times \mathbf{B}$ over all length elements $\mathrm{d}\mathbf{l}$:

$$\mathcal{E} = \oint (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{l} \tag{69}$$

29.5 Induced Electric Fields

non-electrostatic field: an non-conservative field, which cannot have a potential function
induced electric field ($\mathbf{E}$): the non-electrostatic electric field in a stationary closed conducting loop created by a changing magnetic flux
If the path of integration is stationary, then Faraday's law becomes:

$$\mathcal{E} = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \tag{70}$$

29.6 Eddy Currents

eddy currents: induced currents that flow throughout the volume of a material
Applications of them include induction furnaces, eddy current brakes, and metal detectors. Eddy currents also dissipate energy, so transformers are built to avoid them.

29.7 Displacement Current and Maxwell's Equations

displacement current (i_D): the current caused by a changing electric field
displacement current density ($\mathbf{j}_D$): the current density of the displacement current

$$\begin{aligned} i_D &= \epsilon \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \\ \mathbf{j}_D &= \epsilon \frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t} \end{aligned} \tag{71}$$

If the instantaneous actual current i and displacement current i_D are added together, then Ampere's law holds even for time varying electric fields:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu \left(i + i_D\right)_{encl} \tag{72}$$

Maxwell's equations: the four equations that completely describe classical electromagnetism at a microscopic level, where $\epsilon = \epsilon_0$ and $\mu = \mu_0$:

$$\begin{aligned} \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} &= \Phi_E = \frac{Q_{encl}}{\epsilon_0} \\ \oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} &= \Phi_B = 0 \\ \oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} &= -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \\ \oint \mathbf{B} \cdot \mathrm{d}\mathbf{l} &= \mu_0 \left(i_{encl} + \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t}\right) \end{aligned} \tag{73}$$

29.8 Superconductivity

superconductor: a conductor that has zero resistance below the critical temperature T_C when the external magnetic field is below the critical field B_C for that temperature
Meissner effect: the process by which the magnetic field in a superconductor disappears when cooled below the critical temperature
persistent current: an induced current that continues to flow since the magnetic flux through a superconducting loop never changes

30 Inductance

30.1 Mutual Inductance

A changing current flowing in one coil induces an emf in a neighbouring unconnected coil.
mutual inductance (M): the proportionality constant between the number of turns in one coil times the magnetic flux through each turn, and the current in the other coil:

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2} \tag{74}$$

$$\mathcal{E}_2 = -N \frac{\mathrm{d}\Phi_{B1}}{\mathrm{d}t} = -M \frac{\mathrm{d}i_1}{\mathrm{d}t} \tag{75}$$

Henry: SI unit of inductance, $1\,\mathrm{H} = 1\,\mathrm{Wb} \cdot \mathrm{A}^{-1}$

30.2 Self-Inductance and Inductors

self-induced emf: an induced emf in a conductor created by its own changing current
self-inductance (L): the mutual inductance of a coil with itself

$$\begin{aligned} L &= \frac{N\Phi_B}{i} \\ \mathcal{E} &= -L\frac{\mathrm{d}i}{\mathrm{d}t} \end{aligned} \tag{76}$$

inductor or choke: a circuit element designed to have a specific inductance
Although the electric field of a circuit with an inductor is not conservative, if the inductor's coils have negligible resistance, then the inductor produces an electric field opposite to the conservative field by accumulating a voltage V across its terminals. In Kirchhoff's loop rule, an inductor's voltage when travelling against the current is, by Faraday's law:

$$V = L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{77}$$

Applications of inductors include spark coils, fluorescent lights, traffic sensors, and transformers.

30.3 Magnetic-Field Energy

The change in potential energy between an inductor that has no current through it and an inductor that has current I through it is:

$$\Delta U = \frac{1}{2}LI^2 \tag{78}$$

The potential energy in any inductor is stored in the magnetic field of the space inside its coils:

$$u_B = frac12\mu B^2 \tag{79}$$

30.4 The R-L Circuit

R-L **circuit**: a circuit that includes a resistor and an inductor
In a R-L circuit charged by an emf source $\mathcal{E}$, the time constant τ and maximum current I_0 are:

$$\begin{aligned} \tau &= \frac{L}{R} \\ I_0 &= \frac{\mathcal{E}}{R} \end{aligned} \tag{80}$$

When increasing the current, use (45) for i. When decreasing the current, use (44) for i.

30.5 The L-C Circuit

L-C **circuit**: a circuit that includes an inductor and a capacitor
electrical oscillation: the electrical equivalent of mechanical oscillation
phase: the fractional location in a cycle of an oscillation
phase constant (ϕ): the difference in phase between two oscillations
angular frequency (ω): the rate of change of the phase of an oscillation

$$\omega = \frac{1}{\sqrt{LC}} \tag{81}$$

L-C circuits undergo electrical oscillation as energy moves between the inductor and the capacitor. At $t = 0$, if the capacitor is fully charged, then $\phi = 0$; if the capacitor is fully uncharged, then $\phi = \pi/2$. In a L-C circuit with inductance L, capacitance C, and maximum capacitor charge Q_0, the instantaneous current i and capacitor charge q are:

$$\begin{aligned} q &= Q_0 \cos{(\omega t + \phi)} \\ i &= -\omega Q_0 \sin{(\omega t + \phi)} \\ i &= \pm\omega\sqrt{Q_0^2 - q^2} \\ \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C} &= \frac{1}{2}\frac{Q_0^2}{C} \end{aligned} \tag{82}$$

30.6 The L-R-C Series Circuit

L-R-C **series circuit**: a circuit with an inductor, resistor, and capacitor connected in series
damped harmonic motion: oscillation that eventually dies out
L-R-C series circuits undergo damped harmonic motion as energy dissipates through the resistor. At a resistance R, the circuit is **critically damped**; with smaller R the circuit is **underdamped** and oscillates; with larger R the circuit is **overdamped** and returns to equilibrium more slowly. In an underdamped L-R-C series circuit with inductance L, capacitance C, resistance R and maximum charge Q_0, the damped angular frequency ω' and instantaneous charge q are:

$$\begin{aligned} \omega' &= \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad R < \sqrt{4L/C} \\ q &= Q_0 e^{-(R/2L)t} \cos{(\omega' t + \phi)} \end{aligned} \tag{83}$$

31 Alternating Current

31.1 Phasors and Alternating Currents

alternating-current circuits: circuits in which the current changes direction
Since energy losses are inversely proportional to voltage squared, power lines have very high voltages. These voltages must be stepped down for safe consumer use, but transformers can only do so with alternating current, so most power distribution systems use alternating current.
ac source: a circuit element that supplies a sinusoidally varying current or voltage
alternator: an ac source that converts mechanical energy to electrical energy
frequency (f): the number of oscillations per unit time
Commercial power distribution systems in Canada and the United States use $f = 60\,\mathrm{H}z$; most of the rest of the world use $f = 50\,\mathrm{H}z$. Frequency is directly related to angular frequency ω:

$$f = \frac{\omega}{2\pi} \tag{84}$$

voltage amplitude (V): the maximum voltage in an ac circuit
current amplitude (I): the maximum current in an ac circuit
The instantaneous voltage v and current i of an ac source are:

$$\begin{aligned} v &= V\cos\omega t \\ i &= I\cos\omega t \end{aligned} \tag{85}$$

phasor diagram: a geometric tool to help describe ac circuits
Phasor diagrams contain **phasors**, which are vectors that rotate counterclockwise at ω with magnitude V or I. The projection of the phasor on the horizontal axis at an instant is v or i.
rectified average current (I_{rav}): the average current that would flow from a full-wave rectifier, which only allows current in one direction, if the original current passed through it
root-mean-square current (I_{rms}): the square root of the mean of the squared current
Voltages and currents in power distribution systems always use root-mean-square values.
For a sinusoidally varying current or voltage:

$$\begin{aligned} I_{rav} &= \frac{2}{\pi}I = 0.637I \\ I_{rms} &= \frac{1}{\sqrt{2}}I = 0.707I \\ V_{rms} &= \frac{1}{\sqrt{2}}V = 0.707V \end{aligned} \tag{86}$$

31.2 Resistance and Reactance

phase angle (ϕ): the phase constant between current and voltage
The phase angle gives voltage relative to current, so if $i = I\cos\omega t$, then $v = V\cos(\omega t + \phi)$.
inductive reactance (X_L): the tendency for an inductor to oppose changes in current
capacitive reactance (X_C): the tendency for a capacitor to oppose changes in voltage
The reactances for an inductor with inductance L and a capacitor with capacitance C are:

$$\begin{aligned} X_L &= 1\omega L \\ X_C &= \frac{1}{\omega C} \end{aligned} \tag{87}$$

Inductive reactance increases as frequency increases, so inductors are used in **low-pass filters** that block high frequencies. Capacitive reactance increases as frequency decreases, so capacitors are used in **high-pass filters** that block low frequencies and direct current.
In an ac circuit, the voltages across resistors v_R, inductors v_L, and capacitors v_C are:

$$\begin{aligned} v_R &= IR\cos(\omega t) \\ v_L &= IX_L\cos\left(\omega t + \frac{\pi}{2}\right) \\ v_C &= IX_C\cos\left(\omega t - \frac{\pi}{2}\right) \end{aligned} \tag{88}$$

31.3 The L-R-C Series Circuit

impedance (Z): the proportionality constant between V and I

$$Z = \frac{V}{I} \tag{89}$$

In a L-R-C series circuit with an ac source, the current is the same everywhere, and the phasor of the source voltage is the vector sum of the phasors of the voltages of the circuit elements:

$$\begin{aligned} Z^2 &= R^2 + (X_L - X_C)^2 \\ \tan\phi &= \frac{X_L - X_C}{R} \end{aligned} \tag{90}$$

$R = 0$ with no resistor, $L = 0$ with no inductor, and $C = \infty$ with no capacitor.
transients: additional voltages and currents when an ac source is first connected

31.4 Power in Alternating-Current Circuits

The instantaneous power to each circuit element is $p = vi$, but capacitors and inductors are out of phase so that the integral of vi over one cycle is zero. The average power P_{av} for a resistor is:

$$P_{av} = \frac{1}{2}VI = V_{rms}I_{rms} \tag{91}$$

power factor ($\cos\phi$): the proportionality constant between P_{av} and $V_{rms}I_{rms}$

$$\begin{aligned} p = vi &= [V\cos(\omega t + \phi)][I\cos\omega t] \\ P_{av} &= \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi \\ \cos\phi &= \frac{P_{av}}{V_{rms}I_{rms}} = \frac{R}{Z} \end{aligned} \tag{92}$$

31.5 Resonance in Alternating-Current Circuits

resonance: the peaking of the amplitude at a certain frequency
resonance frequency (f_0): the frequency at which the resonance peak occurs
resonance angular frequency (ω_0): the angular frequency at which the resonance peak occurs
The magnitude of the current amplitude peak depends only on resistance:

$$I = \frac{V}{Z} = \frac{V}{R} \tag{93}$$

response curve or resonance curve: a graph of current amplitude versus angular frequency
Current amplitude peaks when there is no voltage across any capacitors or inductors:

$$\begin{aligned} \omega_0 L = X_L = X_C &= \frac{1}{\omega_0 C} \\ \omega_0 &= \frac{1}{LC} \end{aligned} \tag{94}$$

31.6 Transformers

transformer: two coils of wire wound around the same magnetic core
windings: the coils in a transformer, which are insulated from each other
The primary winding induces a emf in the secondary winding by mutual inductance.
In a step-up transformer, the number of turns in the secondary winding $N_2 > N_1$, and in a step-down transformer, $N_1 > N_2$. Transformers can step up or step down voltages accordingly:

$$\begin{aligned} \frac{V_2}{V_1} &= \frac{N_2}{N_1} \\ V_1 I_1 &= V_2 I_2 \end{aligned} \tag{95}$$

The power supplied by a source to a circuit element is greatest when their impedances are equal, and transformers can provide impedance matching to change the impedance of networks.
Energy is always lost in real transformers, from heating, eddy currents, and hysteresis losses. These can be reduced by using a laminated core with a narrow hysteresis loop.

32 Electromagnetic Waves

32.1 Maxwell's Equations and Electromagnetic Waves

electromagnetic wave: a disturbance of time-varying electric and magnetic fields
electromagnetic radiation: another name for electromagnetic waves, since they radiate out
electromagnetic spectrum: the range of electromagnetic waves of all frequencies
visible light: the range of the electromagnetic spectrum that humans can see
The electromagnetic spectrum includes, from longest to shortest wavelength: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X rays, and gamma rays.

32.2 Plane Electromagnetic Waves and the Speed of Light

transverse wave: a wave whose disturbances are perpendicular to the direction of propagation
plane wave: an electromagnetic wave such that its electric and magnetic fields are uniform over any plane perpendicular to the direction of propagation
wave front: the plane at the front of the wave, perpendicular to the direction of propagation
Consider a transverse plane wave propagating with a constant speed c, such that all space behind the wave front has a uniform electric field $\mathbf{E}$ perpendicular to a uniform magnetic field $\mathbf{B}$, and all space in front of the wave front has no field. To satisfy Maxwell's Equations, this must have:

$$\begin{aligned} &\mathbf{E} \perp \mathbf{B} \\ &E = cB \\ &B = \epsilon\mu c E \end{aligned} \tag{96}$$

These equations apply to all electromagnetic waves. The direction of propagation is $\mathbf{E} \times \mathbf{B}$.

Electromagnetic waves require no medium. If the electromagnetic wave travels in vacuum, then $\epsilon = \epsilon_0$ and $\mu = \mu_0$, which are both constant, so that c is the constant speed of light in vacuum:

$$c = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 2.99792458 \times 10^8 \text{ m} \cdot \text{s}^{-1} \tag{97}$$

Waves with continuously varying fields are possible by superposing many plane waves together.
linearly polarized: the property of waves whose electric field is always parallel to one axis

32.3 Sinusoidal Electromagnetic Waves

If the magnitudes of the fields of an electromagnetic wave oscillate sinusoidally:

$$E = E_{max}\cos(kx - \omega t) \qquad B = B_{max}\cos(kx - \omega t) \tag{98}$$

The speed of electromagnetic waves in a dielectric is:

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{\sqrt{KK_m}} \tag{99}$$

index of refraction (n): the proportionality constant between c and v

32.4 Energy and Momentum in Electromagnetic Waves

The energy density u in an electromagnetic wave is:

$$u = u_E + u_B = \frac{1}{2}\epsilon E^2 + \frac{1}{2\mu}\left(\frac{E}{c}\right)^2 = \epsilon E^2 \tag{100}$$

Poynting vector (**S**): the magnitude and direction of the energy of an electromagnetic wave that passes through a plane per unit time per unit cross-sectional area

$$\mathbf{S} = \frac{1}{\mu}\mathbf{E} \times \mathbf{B} \tag{101}$$

intensity (I): the magnitude of the average value of the Poynting vector at a point

$$I = \frac{E_{max}B_{max}}{2\mu} \tag{102}$$

radiation pressure (p_{rad}): the pressure created by the absorption of electromagnetic waves
For a wave totally absorbed by a surface perpendicular to its direction of propagation:

$$p_{rad} = \frac{I}{c} \tag{103}$$

32.5 Standing Electromagnetic Waves

standing wave: the superposition of an incident wave and its reflection
If a wave strikes normal to a conducting sheet, then since **E** must be zero at the conductor:

$$E = -2E_{max}\sin(kx)\sin(\omega t) \qquad B = -2B_{max}\cos(kx)\cos(\omega t) \tag{104}$$

nodal planes: the planes on which a field is always zero
antinodal planes: the planes on which a field can reach its maximum possible value
The nodal planes are half a wavelength apart, and the antinodal planes are halfway between the nodal planes. The nodal planes of the electric field are the antinodal planes of the magnetic field.